LU FACTORIZATION

CS111 · SECTION B · SESSION 3

LU FACTORIZATION WITH PARTIAL PIVOTING

Suppose we want to find the LU decomposition $PA = LU$ for the following matrix using partial pivoting:

$$A = \begin{pmatrix} 1 & 2 & -3 & 4 \\ 4 & 8 & 12 & -8 \\ 2 & 3 & 2 & 1 \\ -3 & -1 & 1 & -4 \end{pmatrix}$$

1	2	-3	4	0
4	8	12	-8	1
2	3	2	1	2
-3	-1	1	-4	3

$\underbrace{\qquad\qquad\qquad}_{A}$ $\underbrace{\quad}_{p}$

$\xrightarrow{\text{Swap } r_0 \text{ and } r_1}$

4	8	12	-8	1
1	2	-3	4	0
2	3	2	1	2
-3	-1	1	-4	3

LU FACTORIZATION WITH PARTIAL PIVOTING

4	8	12	-8		1
1	2	-3	4		0
2	3	2	1		2
-3	-1	1	-4		3

*from previous slide …

Annihilate below (0,0) →

4	8	12	-8		1
$\frac{1}{4}$	0	-6	6		0
$\frac{1}{2}$	-1	-4	5		2
$-\frac{3}{4}$	5	10	-10		3

Swap r_1 and r_3 →

4	8	12	-8		1
$-\frac{3}{4}$	5	10	-10		3
$\frac{1}{2}$	-1	-4	5		2
$\frac{1}{4}$	0	-6	6		0

Annihilate below (1,1) →

4	8	12	-8		1
$-\frac{3}{4}$	5	10	-10		3
$\frac{1}{2}$	$-\frac{1}{5}$	-2	3		2
$\frac{1}{4}$	0	-6	6		0

Swap r_2 and r_3 →

4	8	12	-8		1
$-\frac{3}{4}$	5	10	-10		3
$\frac{1}{4}$	0	-6	6		0
$\frac{1}{2}$	$-\frac{1}{5}$	-2	3		2

LU FACTORIZATION WITH PARTIAL PIVOTING

4	8	12	-8		1
$-\frac{3}{4}$	5	10	-10		3
$\frac{1}{4}$	0	-6	6		0
$\frac{1}{2}$	$-\frac{1}{5}$	-2	3		2

*from previous slide …

Annihilate below (2,2) →

4	8	12	-8		1
$-\frac{3}{4}$	5	10	-10		3
$\frac{1}{4}$	0	-6	6		0
$\frac{1}{2}$	$-\frac{1}{5}$	$\frac{1}{3}$	1		2

Therefore,

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 & 0 \\ \frac{1}{4} & 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix}, \qquad U = \begin{pmatrix} 4 & 8 & 12 & -8 \\ 0 & 5 & 10 & -10 \\ 0 & 0 & -6 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$