

A decorative graphic on the left side of the slide consists of a network of thin, light blue lines. These lines form a complex, branching pattern that resembles a circuit board or a neural network. Some lines are vertical, while others branch out at various angles. Small circles, also in light blue, are placed at various points along the lines, particularly at the ends of branches and at intersections. The overall effect is a modern, technical aesthetic.

LU FACTORIZATION

CS111 • SECTION B • SESSION 3

LU FACTORIZATION WITH PARTIAL PIVOTING

Suppose we want to find the LU decomposition $PA = LU$ for the following matrix using partial pivoting:

$$A = \begin{pmatrix} 1 & 2 & -3 & 4 \\ 4 & 8 & 12 & -8 \\ 2 & 3 & 2 & 1 \\ -3 & -1 & 1 & -4 \end{pmatrix}$$

1	2	-3	4		0
4	8	12	-8		1
2	3	2	1		2
-3	-1	1	-4		3

$\underbrace{\hspace{10em}}_A \quad \underbrace{\hspace{1em}}_p$

Swap r_0 and r_1

4	8	12	-8		1
1	2	-3	4		0
2	3	2	1		2
-3	-1	1	-4		3

LU FACTORIZATION WITH PARTIAL PIVOTING

4	8	12	-8	1
1	2	-3	4	0
2	3	2	1	2
-3	-1	1	-4	3

*from previous slide ...

Annihilate
below
(0,0)

4	8	12	-8	1
$\frac{1}{4}$	0	-6	6	0
$\frac{1}{2}$	-1	-4	5	2
$-\frac{3}{4}$	5	10	-10	3

Swap r_1
and r_3

4	8	12	-8	1
$-\frac{3}{4}$	5	10	-10	3
$\frac{1}{2}$	-1	-4	5	2
$\frac{1}{4}$	0	-6	6	0

Annihilate
below
(1,1)

4	8	12	-8	1
$-\frac{3}{4}$	5	10	-10	3
$\frac{1}{2}$	$-\frac{1}{5}$	-2	3	2
$\frac{1}{4}$	0	-6	6	0

Swap r_2
and r_3

4	8	12	-8	1
$-\frac{3}{4}$	5	10	-10	3
$\frac{1}{4}$	0	-6	6	0
$\frac{1}{2}$	$-\frac{1}{5}$	-2	3	2

LU FACTORIZATION WITH PARTIAL PIVOTING

4	8	12	-8		1
$-\frac{3}{4}$	5	10	-10		3
$\frac{1}{4}$	0	-6	6		0
$\frac{1}{2}$	$-\frac{1}{5}$	-2	3		2

Annihilate below (2,2) →

4	8	12	-8		1
$-\frac{3}{4}$	5	10	-10		3
$\frac{1}{4}$	0	-6	6		0
$\frac{1}{2}$	$-\frac{1}{5}$	$\frac{1}{3}$	1		2

*from previous slide ...

Therefore,

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 & 0 \\ \frac{1}{4} & 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 4 & 8 & 12 & -8 \\ 0 & 5 & 10 & -10 \\ 0 & 0 & -6 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$