Discussion Session 7

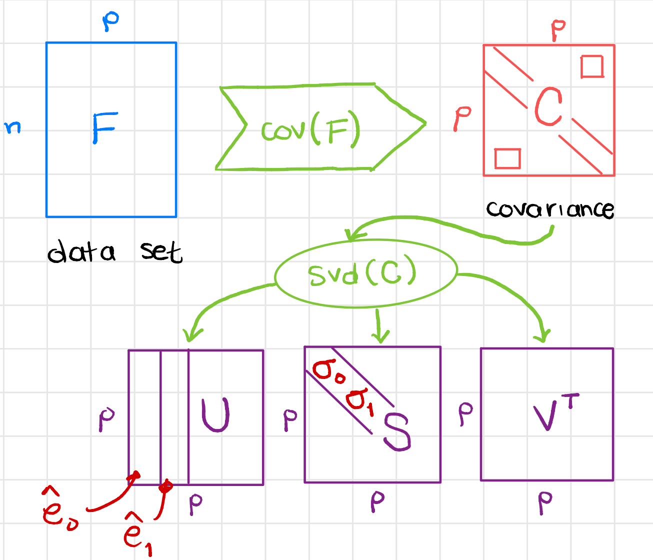
EIGENFACES

Wednesday, Nov 10, 2021

Recall: PCA: Principal Component Analysis.

$\frac{1}{n-1}(F-M)^T(F-M)$

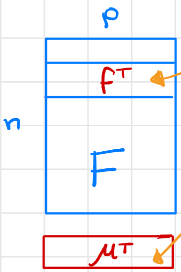
symmetric positive semidefinite.



$U = V$

- The goal is to find the axes on which data varies the most.
- Those axes are the principal components: the singular vectors in U (or V) associated with the largest singular values.

- We can approximate any row f^T in F using the first k principal components (i.e., first k columns of V)



$$f \approx \mu + c_0 \hat{e}_0 + c_1 \hat{e}_1 + \cdots + c_{k-1} \hat{e}_{k-1}, \qquad k \leq p$$

$$c_i = \hat{e}_i^T x \qquad x = f - \mu$$

More compactly:

$$f^{(k)} = \mu + V^{(k)} \mathbf{c}^{(k)} \quad \text{and} \quad \mathbf{c}^{(k)} = [V^{(k)}]^T x$$

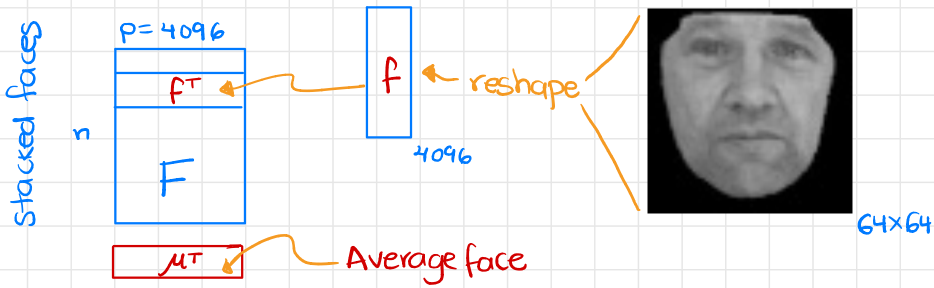
$$\underbrace{f^{(k)}}_{p \times 1} = \underbrace{\mu}_{p \times 1} + \underbrace{V^{(k)}}_{p \times k} \underbrace{\mathbf{c}^{(k)}}_{k \times 1}$$

$$\underbrace{\mathbf{c}^{(k)}}_{k \times 1} = \underbrace{\begin{bmatrix} \hat{e}_0^T \\ \vdots \\ \hat{e}_{k-1}^T \end{bmatrix}}_{\substack{[V^{(k)}]^T \\ k \times p}} \underbrace{x}_{p \times 1}$$

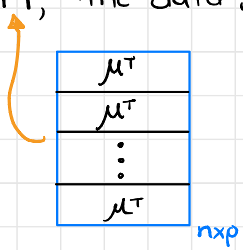
Recall: Each c_i is the coordinate of x with respect to the axis $\hat{e}_i$

Eigenfaces or PCA of a Face Data Set

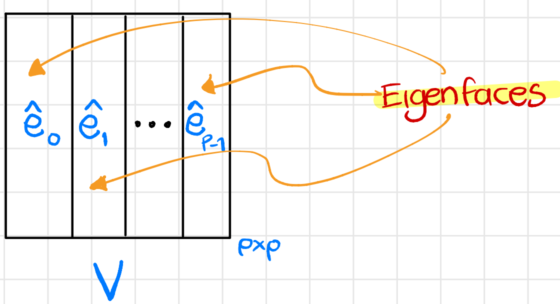
- Build the face data set



- Given F, construct $X = F - M$, the data set centered around the mean face.



- Compute the covariance matrix $C = \frac{1}{n-1} X^T X$.
- Compute the singular value decomposition of $C = USV^T$

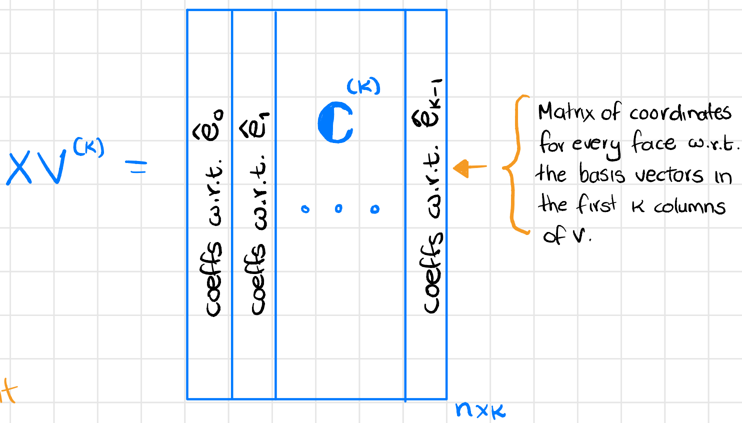


Using Eigenfaces to Generate a Random Face

Goal	Generate a coefficient vector $\tilde{\mathbf{c}}^{(k)}$ that follows the same probability distribution as the samples in F.

- If $X = F - M$, then $\mathbf{C}^{(k)} = XV^{(k)}$ is the $n \times k$ matrix of coeffs for the centered data set X.

- Compute $\mu_{\mathbf{c}}$ and $\sigma^2_{\mathbf{c}}$, the coefficients column mean and variance:



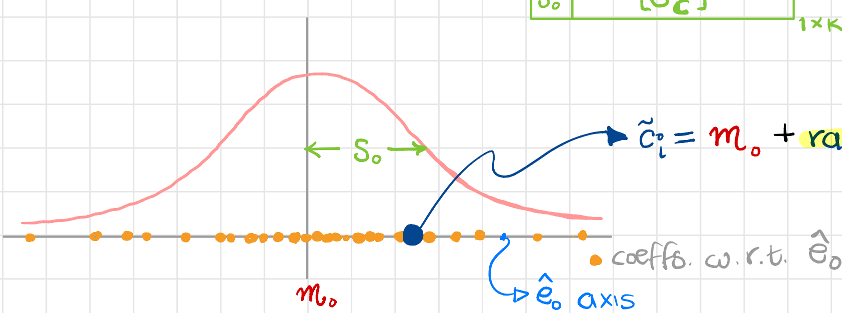
m_0	$\mu^T_{\mathbf{c}}$

$1 \times k$

s_0^2	$[\sigma^2_{\mathbf{c}}]^T$

$1 \times k$

To generate a random coefficient for $\hat{e}_0$, follow a normal distribution $\mathcal{N}(m_0, s_0)$



$\tilde{c}_i = m_0 + \text{randn}(1) * s_0$

- Generate all k coefficients as $\tilde{c}_i = m_i + \text{randn}(1) * s_i$, $0 \leq i \leq k-1$.