FUNCTIONAL DEPENDENCIES AND NORMAL FORMS

DISCUSSION SESSION 10, SECTION A

FINDING THE KEYS BY USING FDS

- X is a **superkey** of R if $X^+ = R$.
- X is a **key** of R iff X is a superkey and no proper subset of X is a superkey. That is, X is **minimal**.

Observations

1. Any candidate key must contain attributes that have not appeared on the right-hand side (RHS) of any functional dependency (FD).
2. If an attribute has occurred on the RHS of some FD, but not on the left-hand side (LHS) of any FD, then it cannot be in any candidate key.

BOYCE-CODD NORMAL FORM

Let R be a relation schema, F be the set of FDs that hold over R, X be a subset of the attributes of R, and A be an attribute of R. R is in **BCNF** if, for every FD $X \rightarrow A$ in F, one of the following statements is true:

- $A \in X$; that is, it is a trivial FD, or
- X is a superkey

THIRD NORMAL FORM

Let R be a relation schema, F be the set of FDs that hold over R, X be a subset of the attributes of R, and A be an attribute of R. R is in **3NF** if, for every FD $X \rightarrow A$ in F, one of the following statements is true:

- $A \in X$; that is, it is a trivial FD, or
- X is a superkey
- A is part of some key for R

EXERCISE 1

Consider the relation $R(ABCDE)$; you are given the following dependencies $F = \{A \to B, BC \to E, ED \to A\}$:

List the keys of R.

Any key must contain CD
$\{ACD\}^+ = \{ABCDE\}$, **OK**.
$\{BCD\}^+ = \{ABCDE\}$, **OK**.
$\{ECD\}^+ = \{ABCDE\}$, **OK**.

$\therefore$ the keys are ACD, BCD, and ECD.

Is R in 3NF?

$A \to B$, non-trivial, $B \in \{BCD\}$; **OK**.
$BC \to E$, non-trivial, $E \in \{ECD\}$; **OK**.
$ED \to A$, non-trivial, $A \in \{ACD\}$; **OK**.

$\therefore$ R is in 3NF.

Is R in BCNF?

$A \to B$, non-trivial, and A is not a superkey of R.
$\therefore$ R is NOT in BCNF.

EXERCISE 2

Suppose R has four attributes $(ABCD)$. Assume that the only dependencies that hold in R are $F = \{AB \rightarrow CD, C \rightarrow A, D \rightarrow B\}$:

Identify the keys for R.

$\{AB\}^+ = \{ABCD\}$, **OK**.
$\{CD\}^+ = \{ABCD\}$, **OK**.
$\{AD\}^+ = \{ABCD\}$, **OK**.
$\{BC\}^+ = \{ABCD\}$, **OK**.

$\therefore$ the keys are AB, CD, AD, and BC.

Identify the best normal form that R satisfies?

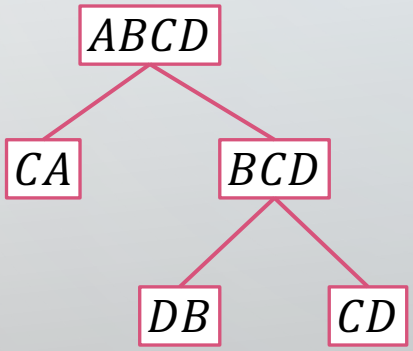
BCNF? **No**: $C \rightarrow A$ and $D \rightarrow B$ violate the norm.
3NF? **Yes**: A, B, C, and D are part of some key.

$\therefore$ R is in 3NF (and 2NF and 1NF).

EXERCISE 2

Suppose R has four attributes $(ABCD)$. Assume that the only dependencies that hold in R are $F = \{AB \rightarrow CD, C \rightarrow A, D \rightarrow B\}$:

Decompose R into a set of BCNF relations that *preserve dependencies*.



Any BCNF decomposition of R doesn't preserve dependencies!

LOSSLESS JOIN AND DEPENDENCY PRESERVING DECOMPOSITIONS

Lossless Join Decomposition

Let R be a relation and F be a set of FDs that hold over R. The decomposition of R into relations with attribute sets R_1 and R_2 is **lossless** if and only if F^+ contains either the FD $R_1 \cap R_2 \rightarrow R_1$ or the FD $R_1 \cap R_2 \rightarrow R_2$.

Dependency-Preserving Decomposition

Let the set of FDs $F_i \subseteq F^+$ be the projection of F onto R_i, where F_i includes only attributes in R_i. Also, let

$$F' = \bigcup_{i=1}^{n} F_i$$

where in general $F' \neq F$.

A **dependency-preserving decomposition** has the property that $(F')^+ = F^+$.

We can check that each FD in $(F - F')$ is logically implied by F'.

EXERCISE 3

Is the following decomposition of $R(ABCDEG)$, with $F = \{AB \to C; AC \to B; AD \to E; B \to D; BC \to A; E \to G\}$,
(a) dependency-preserving? (b) lossless join?

$R_1(AB)$, $R_2(BC)$, $R_3(ABDE)$, $R_4(EG)$

Dependency-preserving?

$F_1 = F_{AB} = \emptyset$.
$F_2 = F_{BC} = \emptyset$.
$F_3 = F_{ABDE} = \{AD \to E; B \to D; AB \to D\}$.
$F_4 = F_{EG} = \{E \to G\}$.

Then, $F' = \{AD \to E; B \to D; E \to G; AB \to D\} \neq F \cup \{AB \to D\}$, and we **can't recover** $AB \to C$ (among others).

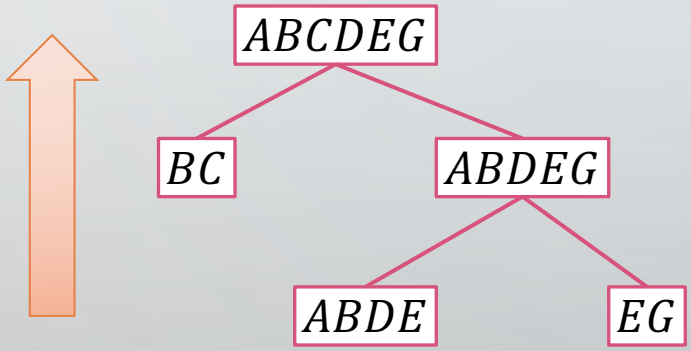
$\therefore$ it's **NOT** a dependency-preserving decomposition.

EXERCISE 3

Is the following decomposition of $R(ABCDEG)$, with $F = \{AB \rightarrow C; AC \rightarrow B; AD \rightarrow E; B \rightarrow D; BC \rightarrow A; E \rightarrow G\}$,
(a) dependency-preserving? (b) lossless join?

$R_1(AB)$, $R_2(BC)$, $R_3(ABDE)$, $R_4(EG)$

Lossless decomposition?



Not a lossless join decomposition
because $B \nrightarrow ADEG$ nor $B \nrightarrow C$.

MINIMAL COVER

Steps to compute the minimal cover, G, of a set of FDs, F:

1. Use the **decomposition** FD inference rule:

   If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$.

2. For $X \rightarrow A$, where $X = Y \cup B$:

   Attribute B is **extraneous** iff $A \in Y^+$ w.r.t. F.

3. $X \rightarrow A$ is **redundant** iff the set $F - \{X \rightarrow A\} \equiv F$:

   That is, compute X^+ w.r.t. $F - \{X \rightarrow A\}$,

   $X \rightarrow A$ is redundant iff $A \in X^+$.

EXERCISE 4

Consider the following set F of FDs on the schema $R(ABC)$:

$$A \rightarrow BC$$
$$B \rightarrow C$$
$$A \rightarrow B$$
$$AB \rightarrow C$$

Compute the minimal cover, G, of F.

Step 1: Decompose

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow C$
(4) $AB \rightarrow C$

Step 2: Remove extraneous attributes.

Is B extraneous in (4)? **Yes!**
We have that $A^+ = \{A, B, C\}$, then, $C \in A^+$.

$\therefore$

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow C$

Step 3: Remove redundant FDs.

(2) $A \rightarrow C$ is redundant because it's implied by (1) and (3). Hence:

$$G = \begin{Bmatrix} A \rightarrow B \\ B \rightarrow C \end{Bmatrix}$$

EXERCISE 5

Consider the following set of FDs on the schema $R(ABCDEH)$:

$$A \rightarrow BCD$$
$$BC \rightarrow DE$$
$$B \rightarrow D$$
$$D \rightarrow A$$

(a) Compute B^+.

$$\boxed{B^+ = \{A, B, C, D, E\}}$$

EXERCISE 5

Consider the following set of FDs on the schema $R(ABCDEH)$:

$$A \rightarrow BCD$$
$$BC \rightarrow DE$$
$$B \rightarrow D$$
$$D \rightarrow A$$

(b) Compute the minimal cover of the above set of FDs.

Step 1:

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $A \rightarrow D$
(4) $BC \rightarrow D$
(5) $BC \rightarrow E$
(6) $B \rightarrow D$
(7) $D \rightarrow A$

Step 2:

Since $B^+ = \{A, B, C, D, E\}$, $\{D, E\} \in B^+ \Longrightarrow C$ is extraneous in (4) and (5).

$\therefore$

(1) $A \rightarrow B$ (2) $A \rightarrow C$
(3) $A \rightarrow D$ (4) $B \rightarrow E$
(5) $B \rightarrow D$ (6) $D \rightarrow A$

Step 3:

(3) $A \rightarrow D$ is redundant because it's implied by (1) and (5).

$\therefore$

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow E$
(4) $B \rightarrow D$
(5) $D \rightarrow A$

EXERCISE 5

Consider the following set of FDs on the schema $R(ABCDEH)$:

$$A \rightarrow BCD$$
$$BC \rightarrow DE$$
$$B \rightarrow D$$
$$D \rightarrow A$$

(c) Give a 3NF decomposition of R using its minimal cover.

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow E$
(4) $B \rightarrow D$
(5) $D \rightarrow A$

Keys:
H must be in all keys, and E cannot be in any of them.

$\{AH\}^+ = \{ABCDEH\}$, **OK**.
$\{BH\}^+ = \{ABCDEH\}$, **OK**.
$\{DH\}^+ = \{ABCDEH\}$, **OK**.

$\therefore$ the keys are AH, BH, and DH.

Decomposition:
Using 3NF synthesis:

$R_1(ABC)$
$R_2(BED)$
$R_3(DA)$
$R_4(AH)$

EXERCISE 5

Consider the following set of FDs on the schema $R(ABCDEH)$:

$$A \rightarrow BCD$$
$$BC \rightarrow DE$$
$$B \rightarrow D$$
$$D \rightarrow A$$

(c) Give a BCNF decomposition of R using its minimal cover.

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow E$
(4) $B \rightarrow D$
(5) $D \rightarrow A$

Keys: AH, BH, and DH.