FUNCTIONAL DEPENDENCIES AND NORMAL FORMS

DISCUSSION SESSION 10, SECTION A
FINDING THE KEYS BY USING FDS

• $X$ is a **superkey** of $R$ if $X^+ = R$.

• $X$ is a **key** of $R$ iff $X$ is a superkey and no proper subset of $X$ is a superkey. That is, $X$ is **minimal**.

**Observations**

1. Any candidate key must contain attributes that have not appeared on the right-hand side (RHS) of any functional dependency (FD).

2. If an attribute has occurred on the RHS of some FD, but not on the left-hand side (LHS) of any FD, then it cannot be in any candidate key.
Let $R$ be a relation schema, $F$ be the set of FDs that hold over $R$, $X$ be a subset of the attributes of $R$, and $A$ be an attribute of $R$. $R$ is in **BCNF** if, for every FD $X \rightarrow A$ in $F$, one of the following statements is true:

- $A \in X$; that is, it is a trivial FD, or
- $X$ is a superkey
Let $R$ be a relation schema, $F$ be the set of FDs that hold over $R$, $X$ be a subset of the attributes of $R$, and $A$ be an attribute of $R$. $R$ is in 3NF if, for every FD $X \rightarrow A$ in $F$, one of the following statements is true:

- $A \in X$; that is, it is a trivial FD, or
- $X$ is a superkey
- $A$ is part of some key for $R$
EXERCISE 1

Consider the relation \( R(ABCDE) \); you are given the following dependencies \( F = \{ A \rightarrow B, BC \rightarrow E, ED \rightarrow A \} \):

List the keys of \( R \).

Any key must contain \( CD \)
- \( \{ACD\}^+ = \{ABCDE\} \), OK.
- \( \{BCD\}^+ = \{ABCDE\} \), OK.
- \( \{ECD\}^+ = \{ABCDE\} \), OK.

\( \therefore \) the keys are \( ACD, BCD, \) and \( ECD \).

Is \( R \) in 3NF?

\( A \rightarrow B \), non-trivial, \( B \in \{BCD\} \); OK.
\( BC \rightarrow E \), non-trivial, \( E \in \{ECD\} \); OK.
\( ED \rightarrow A \), non-trivial, \( A \in \{ACD\} \); OK.

\( \therefore \) \( R \) is in 3NF.

Is \( R \) in BCNF?

\( A \rightarrow B \), non-trivial, and \( A \) is not a superkey of \( R \).

\( \therefore \) \( R \) is NOT in BCNF.
EXERCISE 2

Suppose $R$ has four attributes $(ABCD)$. Assume that the only dependencies that hold in $R$ are $F = \{AB \rightarrow CD, C \rightarrow A, D \rightarrow B\}$:

Identify the keys for $R$.

- $(AB)^+ = \{ABCD\}$, OK.
- $(CD)^+ = \{ABCD\}$, OK.
- $(AD)^+ = \{ABCD\}$, OK.
- $(BC)^+ = \{ABCD\}$, OK.

$\therefore$ the keys are $AB$, $CD$, $AD$, and $BC$.

Identify the best normal form that $R$ satisfies?

- BCNF? **No**: $C \rightarrow A$ and $D \rightarrow B$ violate the norm.
- 3NF? **Yes**: $A, B, C,$ and $D$ are part of some key.

$\therefore$ $R$ is in 3NF (and 2NF and 1NF).
EXERCISE 2

Suppose $R$ has four attributes $(ABCD)$. Assume that the only dependencies that hold in $R$ are $F = \{AB \rightarrow CD, C \rightarrow A, D \rightarrow B\}$:

Decompose $R$ into a set of BCNF relations that preserve dependencies.

Any BCNF decomposition of $R$ doesn’t preserve dependencies!
**Lossless Join Decomposition**

Let $R$ be a relation and $F$ be a set of FDs that hold over $R$. The decomposition of $R$ into relations with attribute sets $R_3$ and $R_4$ is **lossless** if and only if $F^+$ contains either the FD $R_3 \cap R_4 \rightarrow R_3$ or the FD $R_3 \cap R_4 \rightarrow R_4$.

**Dependency-Preserving Decomposition**

Let the set of FDs $F_i \subseteq F^+$ be the projection of $F$ onto $R_i$, where $F_i$ includes only attributes in $R_i$. Also, let

$$F' = \bigcup_{i=1}^{n} F_i$$

where in general $F' \neq F$.

A **dependency-preserving decomposition** has the property that $(F')^+ = F^+$. We can check that each FD in $(F - F')$ is logically implied by $F'$. 
EXERCISE 3

Is the following decomposition of $R(ABCDEG)$, with $F = \{AB \rightarrow C; AC \rightarrow B; AD \rightarrow E; B \rightarrow D; BC \rightarrow A; E \rightarrow G\}$, (a) dependency-preserving? (b) lossless join?

$R_1(AB), \quad R_2(BC), \quad R_3(ABDE), \quad R_4(EG)$

Dependency-preserving?

$F_1 = F_{AB} = \emptyset.$
$F_2 = F_{BC} = \emptyset.$
$F_3 = F_{ABDE} = \{AD \rightarrow E; B \rightarrow D; AB \rightarrow D\}.$
$F_4 = F_{EG} = \{E \rightarrow G\}.$

Then, $F' = \{AD \rightarrow E; B \rightarrow D; E \rightarrow G; AB \rightarrow D\} \neq F \cup \{AB \rightarrow D\}$, and we can’t recover $AB \rightarrow C$ (among others).

$\therefore$ it’s NOT a dependency-preserving decomposition.
EXERCISE 3

Is the following decomposition of $R(ABCDEG)$, with $F = \{AB \rightarrow C; AC \rightarrow B; AD \rightarrow E; B \rightarrow D; BC \rightarrow A; E \rightarrow G\}$, (a) dependency-preserving? (b) lossless join?

$R_1(AB), \quad R_2(BC), \quad R_3(ABDE), \quad R_4(EG)$

Lossless decomposition?

Not a lossless join decomposition because $B \not\leftrightarrow ADEG$ nor $B \not\leftrightarrow C$. 
MINIMAL COVER

Steps to compute the minimal cover, \( G \), of a set of FDs, \( F \):

1. Use the decomposition FD inference rule:
   If \( X \rightarrow YZ \), then \( X \rightarrow Y \) and \( X \rightarrow Z \).

2. For \( X \rightarrow A \), where \( X = Y \cup B \):
   Attribute \( B \) is extraneous iff \( A \in Y^+ \) w.r.t. \( F \).

3. \( X \rightarrow A \) is redundant iff the set \( F - \{X \rightarrow A\} \equiv F \):
   That is, compute \( X^+ \) w.r.t. \( F - \{X \rightarrow A\} \),
   \( X \rightarrow A \) is redundant iff \( A \in X^+ \).
EXERCISE 4

Consider the following set $F$ of FDs on the schema $R(ABC)$:

- $A \rightarrow BC$
- $B \rightarrow C$
- $A \rightarrow B$
- $AB \rightarrow C$

Compute the minimal cover, $G$, of $F$.

**Step 1:** Decompose

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow C$
(4) $AB \rightarrow C$

**Step 2:** Remove extraneous attributes.

Is $B$ extraneous in (4)? **Yes!**

We have that $A^+ = \{A, B, C\}$, then, $C \in A^+$.

\[ \therefore \]

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow C$

**Step 3:** Remove redundant FDs.

(2) $A \rightarrow C$ is redundant because it's implied by (1) and (3). Hence:

\[ G = \{A \rightarrow B, B \rightarrow C\} \]
EXERCISE 5

Consider the following set of FDs on the schema $R(ABCDXE)$:

- $A \rightarrow BCD$
- $BC \rightarrow DE$
- $B \rightarrow D$
- $D \rightarrow A$

(a) Compute $B^+$.  

$$B^+ = \{A, B, C, D, E\}$$
EXERCISE 5

Consider the following set of FDs on the schema \( R(ABCDEH) \):

\[
\begin{align*}
A & \rightarrow BCD \\
BC & \rightarrow DE \\
B & \rightarrow D \\
D & \rightarrow A
\end{align*}
\]

(b) Compute the minimal cover of the above set of FDs.

**Step 1:**
1. \( A \rightarrow B \)
2. \( A \rightarrow C \)
3. \( A \rightarrow D \)
4. \( BC \rightarrow D \)
5. \( BC \rightarrow E \)
6. \( B \rightarrow D \)
7. \( D \rightarrow A \)

**Step 2:**
Since \( B^+ = \{A, B, C, D, E\} \), \( \{D, E\} \in B^+ \Rightarrow C \) is extraneous in (4) and (5).

\[
\begin{align*}
\text{(1) } A & \rightarrow B \\
\text{(2) } A & \rightarrow C \\
\text{(3) } A & \rightarrow D \\
\text{(4) } B & \rightarrow E \\
\text{(5) } B & \rightarrow D \\
\text{(6) } D & \rightarrow A
\end{align*}
\]

**Step 3:**
(3) \( A \rightarrow D \) is redundant because it’s implied by (1) and (5).

\[
\begin{align*}
\text{(1) } A & \rightarrow B \\
\text{(2) } A & \rightarrow C \\
\text{(3) } B & \rightarrow E \\
\text{(4) } B & \rightarrow D \\
\text{(5) } D & \rightarrow A
\end{align*}
\]
EXERCISE 5

Consider the following set of FDs on the schema \( R(ABCDEH) \):

\[
A \rightarrow BCD \\
BC \rightarrow DE \\
B \rightarrow D \\
D \rightarrow A
\]

(c) Give a 3NF decomposition of \( R \) using its minimal cover.

**Keys:**

\( H \) must be in all keys, and \( E \) cannot be in any of them.

\[
\begin{align*}
\{AH\}^+ &= \{ABCDEH\}, \text{ OK.} \\
\{BH\}^+ &= \{ABCDEH\}, \text{ OK.} \\
\{DH\}^+ &= \{ABCDEH\}, \text{ OK.}
\end{align*}
\]

\[\therefore \text{ the keys are } AH, BH, \text{ and } DH.\]

**Decomposition:**

Using 3NF synthesis:

\[
\begin{align*}
R_1(ABC) \\
R_2(BED) \\
R_3(DA) \\
R_4(AH)
\end{align*}
\]
EXERCISE 5

Consider the following set of FDs on the schema $R(ABCDEH)$:

- $A \rightarrow BCD$
- $BC \rightarrow DE$
- $B \rightarrow D$
- $D \rightarrow A$

(c) Give a BCNF decomposition of $R$ using its minimal cover.

**Keys:** $AH$, $BH$, and $DH$. 

Diagram:
- $ABCDEH$
- $BD$
- $ABCEH$
- $BE$
- $ABCH$
- $AB$
- $ACH$
- $AC$
- $AH$

(1) $A \rightarrow B$
(2) $A \rightarrow C$
(3) $B \rightarrow E$
(4) $B \rightarrow D$
(5) $D \rightarrow A$